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BCS states as squeezed fermion-pair states

W.S. Liu[a] and X.P. Li

Department of Physics, Shanxi University, Taiyuan 030006, P.R. China

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Abstract. A new formulation of the Bogoliubov-Valatin transformation explicitly parameter-dependent and with no quasiparticle operators is derived and applied to a diagonalization of the BCS reduced Hamiltonian in the particle space. It is shown that the eigenstates of this Hamiltonian are just the BCS states and are a kind of squeezed fermion-pair states as well.

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1 Introduction

In some traditional theories of superconductivity the Bogoliubov-Valatin transformation (BVT) [1–3] is applied to diagonalize the BCS reduced Hamiltonian [4,5] in the so-called quasiparticle space, and the energy spectrum and the ground state of the system are obtained in this space. The essential effects associated with superconductivity of the quantum fermion liquid are then explained in the language of quasiparticles or elementary excitations. In contrast with the idea of quasiparticles, the characteristic features of the BCS states, which consist of correlated pairs with an attractive interaction between electrons with opposite momenta and antiparallel spins, seem to be neglected all along. Some time ago Svozil [6] introduced the squeezed fermion states in close anology with the two-photon coherent states [7,8] by virtue of the BVT approach. According to Svozil, the BCS model is thought a good example to generate the intramode squeezing of electro-hole sectors. However, the details of the problem remain vague.

In this paper instead of the formal introduction of squeezed fermion states and the conjecture that the BCS states might be these sort of states, we first construct a new formulation of the BVT explicitly parameter-dependent and with no quasiparticle operators. Using these fomulas we then can diagonalize the BCS reduced Hamiltonian in the particle space rather than the quasiparticle space. One important feature of the new method lies in that the unitary operator performed the transformation and the eigenstates of the Hamiltonian diagonalized can immediately be determinted. In particular, they naturally present a similar appearance to those of the two-mode squeeze operator [9] and squeezed number states [7, 10] of light if the parameter is properly chosen. We will show that these eigenstates are precisely the usual BCS states, and can reasonably be called the squeezed fermion-pair states. Finally, the result of our theory will be compared with the known experimental ones of superconductivity.

[a] e-mail: itpshu@public.ty.sx.cn

2 New fomulation of the BVT

The general BVT which mixes the annihilation operators $a_{\mathbf{p}\uparrow}(a_{-\mathbf{p}\downarrow})$ and the creation operators $a^+_{\mathbf{p}\uparrow}(a^+_{-\mathbf{p}\downarrow})$ of a pair of fermions with opposite momenta $(\mathbf{p},-\mathbf{p})$ and antiparallel spins $(\uparrow,\downarrow)$ may be written in the form of the unitary transformations for the individual fermion operators

$$\begin{aligned} U_p a_{\mathbf{p}\uparrow} U_p^+ &= \mu_p a_{\mathbf{p}\uparrow} - \nu_p a^+_{-\mathbf{p}\downarrow}, \\ U_p a^+_{\mathbf{p}\uparrow} U_p^+ &= \mu_p^* a^+_{\mathbf{p}\uparrow} - \nu_p^* a_{-\mathbf{p}\downarrow}, \\ U_p a_{-\mathbf{p}\downarrow} U_p^+ &= \mu_p a_{-\mathbf{p}\downarrow} + \nu_p a^+_{\mathbf{p}\uparrow}, \\ U_p a^+_{-\mathbf{p}\downarrow} U_p^+ &= \mu_p^* a^+_{-\mathbf{p}\downarrow} + \nu_p^* a_{\mathbf{p}\uparrow} \end{aligned} \tag{1}$$

where U_p is an unitary operator, μ_p and ν_p are complex transformation coefficients. The unitarity of the transformation requires that

$$|\mu_p|^2 + |\nu_p|^2 = 1. \tag{2}$$

Now we assume that all U_p, μ_p, and ν_p are the functions of a certain real parameter x. Differentiating equation (1) with respect to x and using the relation $U_p(\partial U_p^+/\partial x) = -(\partial U_p/\partial x)U_p^+$, we obtain

$$\begin{aligned} [\frac{\partial U_p}{\partial x}U_p^+, U_p a_{\mathbf{p}\uparrow} U_p^+] &= \frac{\partial \mu_p}{\partial x} a_{\mathbf{p}\uparrow} - \frac{\partial \nu_p}{\partial x} a^+_{-\mathbf{p}\downarrow}, \\ [\frac{\partial U_p}{\partial x}U_p^+, U_p a^+_{\mathbf{p}\uparrow} U_p^+] &= \frac{\partial \mu_p^*}{\partial x} a^+_{\mathbf{p}\uparrow} - \frac{\partial \nu_p^*}{\partial x} a_{-\mathbf{p}\downarrow}, \\ [\frac{\partial U_p}{\partial x}U_p^+, U_p a_{-\mathbf{p}\downarrow} U_p^+] &= \frac{\partial \mu_p}{\partial x} a_{-\mathbf{p}\downarrow} + \frac{\partial \nu_p}{\partial x} a^+_{\mathbf{p}\uparrow}, \\ [\frac{\partial U_p}{\partial x}U_p^+, U_p a^+_{-\mathbf{p}\downarrow} U_p^+] &= \frac{\partial \mu_p^*}{\partial x} a^+_{-\mathbf{p}\downarrow} + \frac{\partial \nu_p^*}{\partial x} a_{\mathbf{p}\uparrow} \end{aligned} \tag{3}$$

where the sign [,] denotes a commutator. Substituting equation (1) into (3) and multiplying each expression by $\mu_p(\mu_p^*)$ and $\nu_p(\nu_p^*)$ separately, it follows that

$$\begin{aligned}
[\frac{\partial U_p}{\partial x}U_p^+, a_{\mathbf{p}\uparrow}] &= \kappa_p a_{\mathbf{p}\uparrow} - \zeta_p^* a_{-\mathbf{p}\downarrow}^+, \\
[\frac{\partial U_p}{\partial x}U_p^+, a_{\mathbf{p}\uparrow}^+] &= \kappa_p^* a_{\mathbf{p}\uparrow}^+ - \zeta_p a_{-\mathbf{p}\downarrow}, \\
[\frac{\partial U_p}{\partial x}U_p^+, a_{-\mathbf{p}\downarrow}] &= \kappa_p a_{-\mathbf{p}\downarrow} + \zeta_p^* a_{\mathbf{p}\uparrow}^+, \\
[\frac{\partial U_p}{\partial x}U_p^+, a_{-\mathbf{p}\downarrow}^+] &= \kappa_p^* a_{-\mathbf{p}\downarrow}^+ + \zeta_p a_{\mathbf{p}\uparrow}
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\zeta_p(x) &= \mu_p \frac{\partial \nu_p^*}{\partial x} - \nu_p^* \frac{\partial \mu_p(x)}{\partial x}, \\
\kappa_p(x) &= \mu_p^* \frac{\partial \mu_p}{\partial x} + \nu_p \frac{\partial \nu_p^*(x)}{\partial x} = -\kappa_p^*(x).
\end{aligned} \tag{5}$$

Multiplying again each of equation (4) by the operators $a_{\mathbf{p}\uparrow}(a_{\mathbf{p}\uparrow}^+)$ and $a_{-\mathbf{p}\downarrow}(a_{-\mathbf{p}\downarrow}^+)$ separately and rearranging properly, we obtain

$$\begin{aligned}
&\frac{\partial U_p}{\partial x}U_p^+ - \frac{1}{2}\left(a_{\mathbf{p}\uparrow}^+ \frac{\partial U_p}{\partial x}U_p^+ a_{\mathbf{p}\uparrow} + a_{-\mathbf{p}\downarrow}^+ \frac{\partial U_p}{\partial x}U_p^+ a_{-\mathbf{p}\downarrow}\right. \\
&\qquad \left. + a_{\mathbf{p}\uparrow}\frac{\partial U_p}{\partial x}U_p^+ a_{\mathbf{p}\uparrow}^+ + a_{-\mathbf{p}\downarrow}\frac{\partial U_p}{\partial x}U_p^+ a_{-\mathbf{p}\downarrow}^+\right) \\
&= \zeta_p^* a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+ - \zeta_p a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow} - \kappa_p(a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} + a_{-\mathbf{p}\downarrow}^+ a_{-\mathbf{p}\downarrow} - 1).
\end{aligned} \tag{6}$$

Supposing that the operator $(\partial U_p/\partial x)U_p^+$ is in proportion to the right hand side of equation (6),

$$\begin{aligned}
\frac{\partial U_p}{\partial x}U_p^+ = k[&\zeta_p^*(x)a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+ - \zeta_p(x)a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow} \\
&- \kappa_p(x)(a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} + a_{-\mathbf{p}\downarrow}^+ a_{-\mathbf{p}\downarrow} - 1)]
\end{aligned} \tag{7}$$

where k is a proportion constant to be determined. The substitution of equation (7) back into (6) and the straightforward calculation give that $k = 1$, which then leads to

$$U_p(x) = \exp[\xi_p^*(x)K_+ - \xi_p(x)K_- + i2\eta_p(x)K_0] \tag{8}$$

where

$$\begin{aligned}
K_+ &= a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+, \qquad K_- = a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}, \\
K_0 &= \frac{1}{2}(a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} + a_{-\mathbf{p}\downarrow}^+ a_{-\mathbf{p}\downarrow} - 1)
\end{aligned} \tag{9}$$

are the generators of the $SU(2)$ Lie algebra, which satisfy the commutation relation

$$[K_-, K_+] = -2K_0, \qquad [K_0, K_\pm] = \pm K_0 \tag{10}$$

and

$$\xi_p(x) = \int_{x_0}^{x} \zeta_p(x')\mathrm{d}x', \qquad \eta_p(x) = i\int_{x_0}^{x} \kappa_p(x')\mathrm{d}x' \tag{11}$$

where $\xi_p(x_0) = \eta_p(x_0) = 0$ and $U_p(x_0) = 1$, or equivalently, $\mu_p(x_0) = 1$ and $\nu_p(x_0) = 0$. Moreover, the unitarity of U_p requires that $\xi_p(x)$ has to be a complex function of x and $\eta_p(x)$ a real function of x. These requirements will impose strong restrictions on a choice of x and x_0.

On the other hand, if we perform the transformation (1) directly by using the unitary operator (8), we will have

$$\begin{aligned}
\mu_p(x) &= \cos\Omega_p(x) - \frac{i\eta_p(x)}{\Omega_p(x)}\sin\Omega_p(x), \\
\nu_p(x) &= \frac{\xi_p^*(x)}{\Omega_p(x)}\sin\Omega_p(x)
\end{aligned} \tag{12}$$

where $\Omega_p(x) = [|\xi_p(x)|^2 + \eta_p^2(x)]^{1/2}$. It is evident that equations (5), (11) and (12) form a set of coupled integrodifferential equations with respect to $\xi_p(x)$ and $\eta_p(x)$ or $\mu_p(x)$ and $\nu_p(x)$. These equations must be solved self-consistently. It is easy to see that their self-consistent solutions can readily be found in practical applications if the parameter is ingeniously assigned. This means that equations (1), (5), (8), (11) and (12) would form a set of parameter-dependent, closed expressions of the BVT.

3 Diagonalization of the BCS reduced Hamiltonian

Employing the grand canonical ensemble and a mean field approximation of the interaction, as is usually done in the quasiparticle theory, the BCS reduced Hamiltonian can be written as

$$\begin{aligned}
H' = \sum_{\mathbf{p}>0}\{&\varepsilon_p a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} + \varepsilon_p a_{-\mathbf{p}\downarrow}^+ a_{-\mathbf{p}\downarrow} \\
&+ \Delta_p^* a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow} + \Delta_p a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+ - \Delta_p^* X_p\}
\end{aligned} \tag{13}$$

where $\varepsilon_p = p^2/2m - \mu$ is the energy measured from the Fermi surface and $\Delta_p = \sum_{p'} V_{pp'}X_{p'}$ is the so-called energy gap function. Here μ is the chemical potential, $V_{pp'} = <\mathbf{p}\uparrow, -\mathbf{p}\downarrow|V|\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow>$ represents the matrix element of the attractive interaction between fermion pairs with opposite momenta and spins, and $X_p = <a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}>$ is the ensemble average of the operators within the angle brackets.

Let us further perform the unitary transformation for H' and require that the transformed Hamiltonian is to be diagonal in the fermion number operators, that is

$$UH'U^+ = E_0 + \sum_{\mathbf{p}\neq 0} E_p a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} \tag{14}$$

where $U = \prod_p U_p$ is the total unitary operator, E_0 and E_p are the energy of the ground state and the energy spectrum of the system respectively. The direct calculation

gives:

$$-2\varepsilon_p\mu_p\nu_p^* + (\Delta_p^*\mu_p^2 - \Delta_p\nu_p^{*2}) = 0, \quad (15)$$

$$\varepsilon_p[|\mu_p|^2 - |\nu_p|^2 + \Delta_p^*\mu_p\nu_p + \Delta_p\mu_p^*\nu_p^*] = E_p, \quad (16)$$

$$\sum_{p\neq 0}[\varepsilon_p|\nu_p|^2 - (\Delta_p^*\mu_p\nu_p + \Delta_p\mu_p^*\nu_p^*) - \Delta_p X_p] = E_0. \quad (17)$$

First, the combination of equations (15) and (2) give the solutions of μ and ν to be

$$\mu_p = \left(\frac{\lambda_p+1}{2\lambda_p}\right)^{1/2}, \qquad \nu_p = \left(\frac{\lambda_p-1}{2\lambda_p}\right)^{1/2} \quad (18)$$

where

$$\lambda_p = (1+\alpha_p^2)^{1/2}, \qquad \alpha_p = \frac{\Delta_p}{\varepsilon_p}\cdot \quad (19)$$

It should be noted that we have assumed Δ_p is a real quantity by a suitable choice of phases, so λ_p, μ_p and ν_p are all the real functions of α_p. Since α_p is associated with the attractive potential V *via* Δ_p, and $V_{pp'}$, $\mu_p|_{\alpha_p=0} = 1$ and $\nu_p|_{\alpha_p=0} = 0$ physically correspond to the case of the ideal fermion system when the interaction is equal to zero. Therefore it is appropriate to choose α_p as the parameter x and take $\alpha_p = 0$ to be the lower limit of the integrals in equation (11). Thus

$$\xi_p(\alpha_p) = \frac{1}{2}\alpha_p\ {}_2F_1(\frac{1}{2},1;\frac{3}{2};-\alpha_p^2), \qquad \eta_p(\alpha_p) = 0. \quad (20)$$

where ${}_2F_1$ is the Gaussian hypergeometric function and $\alpha_p < 1$ [11]. From this result equations (8) and (12) then become

$$U_p(\alpha_p) = \exp\{\xi_p(r)[a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+ - a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}]\}. \quad (21)$$

$$\mu_p = \cos\xi_p(\alpha_p), \qquad \nu_p(\alpha_p) = \sin\xi_p(\alpha_p). \quad (22)$$

Next, substituting equation (22) back into equations (16) and (17), we obtain

$$E_p = (\varepsilon_p^2 + \Delta_p^2)^{1/2}, \quad (23)$$

$$E_0 = -\sum_{p\neq 0}(E_p - \Delta_p - \Delta_p X_p). \quad (24)$$

Not at all suprising, equations (23) and (24) appear in the same forms as those of the BCS theory, but here X_p and Δ_p have to be evaluated in the particle space rather than in the quasiparticle space. It is easily shown that the thermodynamics deduced from them is also quite the same as those of the quasiparticle theory [5]. However, the eigenstates of the Hamiltonian (13) now are obviously determined as

$$|\Phi_{\{n_p,n_{-p}\}}(\alpha_p)> = U^+(\alpha_p)|\{n_p,n_{-p}\}> = \prod_p U_p^+(\alpha_p)|n_p,n_{-p}> \quad (25)$$

where $|\{n_p,n_{-p}\}>=|n_{p_1},n_{-p_1}>|n_{p_2},n_{-p_2}>...$ is a multimode fermion-pair state, which is the direct product of various single fermion-pair states or double fermion states $|n_p,n_{-p}>$ in the double Fock space. The ground states and the complete excitation states of the system correspond to the cases of the occupation numbers $n_p = n_{-p} = 0$ and 1 respectively,

$$|\Phi_{\{0_p,0_{-p}\}}(\alpha_p)> = \prod_{\mathbf{p}} U_p^+(\alpha_p)|0_p,0_{-p}> = \prod_{\mathbf{p}}[\cos\xi_p - \sin\xi_p a_{\mathbf{p}\uparrow}^+ a_{-\mathbf{p}\downarrow}^+]|0_p,0_{-p}>, \quad (26)$$

$$|\Phi_{\{\mathbf{p}\uparrow,-\mathbf{p}\downarrow\}}(\alpha_p)> = \prod_{\mathbf{p}} U_p^+(\alpha_p)|1_p,1_{-p}> = \prod_{\mathbf{p}}[\cos\xi_p + \sin\xi_p a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}]|1_p,1_{-p}>. \quad (27)$$

Following the usual method, the fermion-pair state is constructed as

$$|n_p,n_{-p}>=\frac{(a_{-\mathbf{p}\downarrow}^+)^{n_{-p}}(a_{\mathbf{p}\uparrow}^+)^{n_p}}{\sqrt{n_p!}\sqrt{n_{-p}!}}|0_p,0_{-p}>. \quad (28)$$

Hence $a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}|1_p,1_{-p}>=-|0_p,0_{-p}>$ and $a_{\mathbf{p}\uparrow}^+a_{-\mathbf{p}\downarrow}^+|0_p,0_{-p}>=-|1_p,1_{-p}>$. Then equations (26) and (27) are precisely the standard BCS states [3,5]. In view of striking resemblances between the unitary operator (21) and the Caves-Schumaker two-mode squeeze operator of light, we would therefore expect that $U_p(\alpha_p)$ should play the role of the squeeze fermion operator for the pair of the $(\mathbf{p}\uparrow)$- and $(-\mathbf{p}\downarrow)$-mode in the quantum Fermi liquid, while the eigenstates (25) should be the fermion analog of the two-mode squeezed number states of light.

4 Quantum correlation and squeezing of the BCS ground state

We are now in a position to justify our views above. Without loss of generality we restrict ourselves to the BCS ground state. The density matrix representing this pure state is of the form

$$\rho = U^+|\{0_p,0_{-p}\}><\{0_p,0_{-p}\}|U. \quad (29)$$

The reduced density matrix for the $(\mathbf{p}\uparrow)$-mode is obtained by tracing ρ over the $(-\mathbf{p}\downarrow)$-mode, and is

$$\rho_{\mathbf{p}\uparrow} = Tr_{-\mathbf{p}\downarrow}\rho = \prod_p\{\cos^2\xi_p(\alpha_p)|0_p><0_p| + \sin^2\xi_p(\alpha_p)|1_p><1_p|\}. \quad (30)$$

If we set that $\tan\xi_p = \mathrm{e}^{-\beta\boldsymbol{\omega}_p/2}$, thus $\cos^2\xi_p = 1/(1+\mathrm{e}^{-\beta\boldsymbol{\omega}_p})$ and $\sin^2\xi_p = \mathrm{e}^{-\beta\boldsymbol{\omega}_p}/(1+\mathrm{e}^{-\beta\boldsymbol{\omega}_p})$, equation (30) then becomes

$$\rho_{\mathbf{p}\uparrow} = \prod_p \frac{\mathrm{e}^{-\beta\boldsymbol{\omega}_p a_p^+ a_p}}{1+\mathrm{e}^{-\beta\boldsymbol{\omega}_p}}\cdot \quad (31)$$

Similarly, the reduced density matrix for the $(-\mathbf{p}\downarrow)$-mode has

$$\rho_{-\mathbf{p}\downarrow} = \prod_p \frac{\mathrm{e}^{-\beta\boldsymbol{\omega}_p a^+_{-p}a_{-p}}}{1+\mathrm{e}^{-\beta\boldsymbol{\omega}_p}} \tag{32}$$

where $\boldsymbol{\omega}_p = \boldsymbol{\omega}_{-p}$. These states are of the forms usually associated with chaotic or thermal Fermi-Dirac statistics. Thus both the single modes display thermal fluctuation. On the other hand, we see that the density matrix (29) cannot be written in the factorized form

$$\rho = \rho_{\mathbf{p}\uparrow} \bigotimes \rho_{-\mathbf{p}\downarrow}. \tag{33}$$

Then the BCS ground state is certainly correlated [12,13], unless $\xi_p = 0$, that is, the squeeze parameter $\alpha_p = 0$ or the energy gap $\Delta_p = 0$ for the case of the ideal fermion system.

In order to see the squeezing property of the BCS ground state, instead of the linear superposition of the two single modes, we immediately adopt the BCS annihilation and creation operators corresponding to the pairing destruction and creation of electrons [4,5],

$$b_{\mathbf{p}} = a_{-\mathbf{p}\downarrow}a_{\mathbf{p}\uparrow}, \qquad b^+_{\mathbf{p}} = a^+_{\mathbf{p}\uparrow}a^+_{-\mathbf{p}\downarrow}. \tag{34}$$

Then the Hermitian quadrature components of the annihilation operator are introduced by

$$b_{1\mathbf{p}} = \frac{1}{2}(b_{\mathbf{p}} + b^+_{\mathbf{p}}), \qquad b_{2\mathbf{p}} = \frac{1}{2i}(b_{\mathbf{p}} - b^+_{\mathbf{p}}). \tag{35}$$

Whereas the BCS ground state now is rewritten as

$$|\Phi_{\{0_p,0_{-p}\}}(\alpha_p) = \prod_p [\theta_p|0_p, 0_{-p}> + \tau_p|1_p, 1_{-p}>] \tag{36}$$

where θ_p and τ_p are the probability amplitudes for fermion-pair vaccum and excitation in the double Fock space, they usually have to be complex numbers and $|\theta_p|^2 + |\tau_p|^2 = 1$. For $b_{1\mathbf{p}}$ and $b_{2\mathbf{p}}$ we find that the variances in this state are

$$\begin{aligned} \Delta b^2_{1\mathbf{p}} &= \frac{1}{4}(1 - \sin^2 2\xi_p(\alpha_p)\cos^2 \bigtriangledown\phi), \\ \Delta b^2_{2\mathbf{p}} &= \frac{1}{4}(1 - \sin^2 2\xi_p(\alpha_p)\sin^2 \bigtriangledown\phi) \end{aligned} \tag{37}$$

where $\bigtriangledown\phi$ is the relative phase between θ_p and τ_p. Obviously, squeezing can simultaneously occur in both the qudratures for various $\bigtriangledown\phi$. This would arise from the fact that the two modes, the $(\mathbf{p}\uparrow)$- and $(-\mathbf{p}\downarrow)$-mode, become so tightly correlated because of the attractive interaction, that the quadratures of their complicated superposition modes might no longer fluctuate independently.

Similar cases have occured in two-photon nonlinear processes in quantum optics [9,13,14]. For example, for non-degenerate processes the individual signal and idler modes display isotropic, phase-insensitive Gaussian fluctuations analogous to those usually associated with chaotic or thermal Bose-Einstein fields. However, the combined two-mode state, due to the quantum correlation between the signal and idler modes, is a non-classical state of light exhibiting squeezed fluctuations. Moreover, if the initial state is a double vacuum state squeezing would also simultaneously exist in the two time-dependent quadratures for all times $t > 0$.

Since the squeeze parameter α_p physically cannot take any value, for example, in the weak-coupling limit the coupling constant $N(0)V \ll 1$ (here $N(0)$ is the state density at the Fermi surface), for which [15]

$$\Delta(0) = 2\hbar\boldsymbol{\omega}_D \mathrm{e}^{-1/(N(0)V)}. \tag{38}$$

where $\Delta(0)$ is the energy gap at the temperature $T = 0$ and $\hbar\boldsymbol{\omega}_{\mathrm{D}} \simeq |\varepsilon_p|$ is the energy determined by the Debye frequency $\boldsymbol{\omega}_{\mathrm{D}}$. Thus

$$\alpha_p \simeq \frac{\Delta(0)}{\hbar\boldsymbol{\omega}_{\mathrm{D}}} = 2\mathrm{e}^{-1/(N(0)V)}. \tag{39}$$

The experimental values of $N(0)V$ for some metals are $0.18 \sim 0.39$, the corresponding values of α_p then are $0.0074 \sim 0.164$. For such small values of α_p squeezings are rather small. However, as already noted in reference [16], BCS theory is exact in both the weak- and strong-coupling limits. The larger values of $N(0)V$, *i.e.* α_p, should be also possible.

References

1. N.N. Bogoliubov, J. Exp. Theor. Phys. USSR **34**, 58 (1958).
2. D.G. Valatin, Nuovo Cimento **7**, 843 (1958).
3. K. Yosida, Phys. Rev. **111**, 1255 (1958).
4. J. Bardeen, L.N. Cooper, J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
5. J.R. Schrieffer, *Theory of Superconductivity* (Benjamin, New York, 1964).
6. K. Svozil, Phys. Rev. Lett. **65**, 3341 (1990).
7. H.P. Yuen, Phys. Rev. A **13**, 2226 (1976).
8. R. Loudon, P.L. Knight, J. Mod. Opt. **34**, 709 (1987).
9. C.M. Caves, B.L. Schumaker, Phys. Rev. A **31**, 3068 (1985); B.L. Schumaker, C.M. Caves, Phys. Rev. A **31**, 3093 (1985).
10. M.S. Kim, F.A.M. de Oliveira, P.L. Knight, Phys. Rev. A **40**, 2494 (1989).
11. A. Erdélyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill, New York, 1953) Vol. 1.
12. L.E. Ballentine, *Quantum mechanics* (Prentice-Hall, London, 1990) pp. 160-165.
13. S.M. Barnett, P.L. Knight, J. Opt. Soc. Am. B **2**, 467 (1985); J. Mod. Opt. **34**, 841 (1987).
14. B.R. Mollow, R.J. Glauber, Phys. Rev. **160**, 1076 (1967); **160**, 1097 (1967).
15. P.G. de Gennes, *Superconductivity of Metals and alloys* (Addison-Wesley, New York, 1989) pp. 121-125
16. P. Nozières, S. Schmitt-rink, J. Low Temp. Phys. **59**, 195 (1985).